

# About the inclusion of an infinite number of resonances in anomalous decays.

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The extracted value for the  $g_{\omega\rho\pi}^{eff}$  effective coupling from experimental data, considering only the  $\rho$  meson, resumes not only the  $\rho$  meson effect but also all its additional radial excitation modes. By explicitly adding the radial excitations of the  $\rho$  meson, considering a particular form of the spectrum and relations among the couplings, we identify the single  $g_{\omega\rho\pi}$  and the  $\rho$  radial excitations effect in the  $\omega \rightarrow \pi^0\gamma$  decay. We obtain that the individual coupling is in the range  $g_{\omega\rho\pi} = 8.2 - 8.6 \text{ GeV}^{-1}$ , which is about 40% smaller than the effective  $g_{\omega\rho\pi}^{eff}$ . We verify the consistency with the chiral approach in the  $\pi^0 \rightarrow \gamma\gamma$  and  $\gamma^* \rightarrow 3\pi$  processes. Besides the model dependence, our description succeeds in exhibiting how each contribution came into the game. In particular, we show that for the  $\gamma^* \rightarrow 3\pi$  decay, the usual relation  $\mathcal{A}_{\gamma 3\pi}^{VMD} = (3/2)\mathcal{A}_{\gamma 3\pi}^{WZW}$ , encodes all the vector contributions and not only the  $\rho$  meson one. In addition, we find that there is an almost exact (accidental) cancelation between the radial excitations and the contact term contributions.

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## I. INTRODUCTION

The study of the low energy regime of the strong interaction among quarks relies on the use of effective theories. A description based on the effective hadronic degrees of freedom, namely the vector meson dominance approach (VMD), assigns an effective coupling to the hadronic interaction, which must be determined from the experimental information. Based on that approach, the determination of the coupling between the  $\omega$ ,  $\rho$  and  $\pi$  mesons from different observables has been found to lay in a wide range of values from 11.9 to 15.7  $\text{GeV}^{-1}$  [1], pointing out to possible experimental or theoretical problems. The experimental issues, if any, will be settled with the advent of more precise measurements. On the other hand the theoretical treatment considers that, since phase space forbids the three of the mesons to be on-shell, at least one of them must be off-shell, and considers the lowest lying resonance as the only relevant state. That is, the extracted effective strong coupling resumes all possible additional contributions from higher states and not only the  $\omega - \rho - \pi$  interaction. A proper description to include all of them would require the knowledge of the full excited spectrum and their corresponding couplings. Attempts in this direction have been made [2] by invoking the factorizable dual model which turns out to modify all the tree level diagrams by a form factor to account for all the possible off-mass-shell mesons. In this work we propose a model to explicitly include the effect of the radial excitations considering a particular structure of the spectrum and obtain the truly  $g_{\omega\rho\pi}$  coupling from the  $\omega \rightarrow \pi^0\gamma$  decay and verify the consistency with the chiral approach in the  $\pi^0 \rightarrow \gamma\gamma$  and  $\gamma^* \rightarrow 3\pi$  processes.

## II. BASELINE

Let us make a review of the results from the decays when no radial excitations are included. This will be the baseline of the calculation to compare with. The VMD Lagrangian including the  $\rho$ ,  $\pi$  and  $\omega$  mesons can be set as:

$$\begin{aligned} \mathcal{L} = & g_{\rho\pi\pi}\epsilon_{abc}\rho_\mu^a\pi^b\partial^\mu\pi^c + g_{\omega\rho\pi}\delta_{ab}\epsilon^{\mu\nu\lambda\sigma}\partial_\mu\omega_\nu\partial_\lambda\rho_\sigma^a\pi^b \\ & + g_{3\pi}\epsilon_{abc}\epsilon^{\mu\nu\lambda\sigma}\omega_\mu\partial_\nu\pi^a\partial_\lambda\pi^b\partial_\sigma\pi^c + \frac{em_V^2}{g_V}V_\mu A^\mu + \dots \end{aligned} \quad (1)$$

This Lagrangian exhibits only the relevant pieces for this work and should be part of any effective Lagrangian describing these mesons. Terms with higher derivatives and additional terms which allow to preserve gauge invariance are not shown [3]. We have made explicit the notation regarding the couplings and the corresponding fields and, in the last term,  $V$  refers in general to vector mesons and  $A^\mu$  refers to the photon field. Here  $g_V = 2\alpha\sqrt{\pi m_V/3\Gamma_{V \rightarrow l+l^-}}$  ( $l = e, \mu, \tau$ ). On the other hand, the chiral symmetry of the strong interaction dictates that the Wess Zumino Witten anomaly [4] is responsible for the  $\pi^0 \rightarrow \gamma\gamma$  and  $\gamma^* \rightarrow 3\pi$  decays. For our purposes the relevant part of the Lagrangian is given by:

$$\begin{aligned} \mathcal{L}^{WZW} = & \frac{iN_c e^2}{24\pi^2 f_\pi} \epsilon^{\mu\nu\lambda\sigma} \pi^0 F_{\mu\nu} F_{\lambda\sigma} \\ & + \frac{N_c}{3 \times 6} \frac{e}{4\pi^2 f_\pi^3} \epsilon_{abc} \epsilon^{\mu\nu\lambda\sigma} B_\mu \partial_\nu \pi^a \partial_\lambda \pi^b \partial_\sigma \pi^c + \dots, \end{aligned} \quad (2)$$

where  $N_c$  is the number of colors and  $f_\pi = 0.093 \text{ GeV}$ , and we did split the coefficients for the sake of clarity when writing the amplitudes below.

### A. The $\pi \rightarrow \gamma\gamma$ decay

The amplitude for the process has a generic form:

$$\mathcal{M}_{\pi\gamma\gamma} = \epsilon_{\mu\nu\lambda\sigma} k_1^\mu \eta_1^\nu k_2^\lambda \eta_2^\sigma \mathcal{A}_{\pi\gamma\gamma}, \quad (3)$$

where  $k_1$  ( $\eta_1$ ) and  $k_2$  ( $\eta_2$ ) are the photons momenta (polarizations) and  $\mathcal{A}_{\pi\gamma\gamma}$  encodes the model details. In the chiral approach it is given by the WZW, as mentioned above, while in the VMD approach, the decay can be seen as proceeding through the emission of the  $\rho$  and  $\omega$  mesons which eventually decay into photons. Using the above Lagrangians, and taking the zero momentum limit, they correspond to:

$$\mathcal{A}_{\pi\gamma\gamma}^{VMD} = \frac{2e^2 g_{\omega\rho\pi}}{g_\rho g_\omega}; \quad \mathcal{A}_{\pi\gamma\gamma}^{WZW} = \frac{6e^2}{24\pi^2 f_\pi} \quad (4)$$

Matching both descriptions, it is possible to relate the couplings. Namely,

$$|g_{\omega\rho\pi}^{eff}| = |g_{\rho\pi\pi} g_\omega / 8\pi^2 f_\pi|. \quad (5)$$

Note that this parameter, initially considered as due to a single channel ( $\rho$  and  $\omega$ ), when imposed to account for the total effect of the anomaly becomes an *effective coupling resuming all the possible additional contributions*. This observation will be crucial in the interpretation of the corresponding magnitude.

An additional relationship between the  $\rho$  and the pion properties is given by the so-called KSFR relation[5]:

$$g_{\rho\pi\pi} = \frac{m_\rho}{\sqrt{2}f_\pi}. \quad (6)$$

Considering that the KSFR relation, SU(3) symmetry ( $g_\omega = 3g_\rho$ ) and universality condition ( $g_\rho = g_{\rho\pi\pi}$ ) are hold, the effective coupling Eqn. (5) becomes

$$|g_{\omega\rho\pi}^{eff}| = \frac{3m_\rho^2}{16\pi^2 f_\pi^3} = 14.2 \text{ GeV}^{-1}. \quad (7)$$

### B. The $\gamma^* \rightarrow 3\pi$ decay

The amplitude of the process can be written in general as:

$$\mathcal{M} = i\epsilon_{\mu\alpha\beta\gamma} \eta^\mu p_1^\alpha p_2^\beta p_3^\gamma \mathcal{A}_{\gamma 3\pi}, \quad (8)$$

where  $\mathcal{A}_{\gamma 3\pi}$  encodes the details of the model used to describe the process, and  $p_1, p_2, p_3$  are the pions momenta and  $\eta$  is the photon polarization.

The decay of the photon into three pions in the chiral description also has its origin in the WZW anomaly as discussed above. In the VMD approach, it has been shown that this decay is mainly produced through the  $\omega$  into  $\rho\pi$  decay channel [6], followed by the break down of

the  $\rho$  into another two pions. At zero momentum they correspond to:

$$\mathcal{A}_{\gamma 3\pi}^{WZW} = \frac{e}{4\pi^2 f_\pi^3} \quad \mathcal{A}_{\gamma 3\pi}^{VMD} = \frac{6e}{g_\omega} \frac{g_{\omega\rho\pi} g_{\rho\pi\pi}}{m_\rho^2} \quad (9)$$

(a factor of 6 arises from the momenta permutations when bringing the amplitude to Eqn.8 form)

By linking the anomaly term to the corresponding amplitude from VMD, the KSFR relation is not hold anymore. The reason lies on the fact that, although the  $\rho$  channel approach is well motivated, it is unable to fully capture the anomaly information. i.e. the decay have additional axial contributions which can not be captured in the effective vector channel. Thus, in the VMD description, an additional contact interaction between the  $\omega$  and the  $3\pi$  mesons must be added:

$$\mathcal{L}_{\omega\pi\pi\pi}^c = g_{3\pi}^c \epsilon_{abc} \epsilon^{\mu\nu\lambda\sigma} \omega_\mu \partial_\nu \pi^a \partial_\lambda \pi^b \partial_\sigma \pi^c \quad (10)$$

where  $g_{3\pi}^c$  is the corresponding effective coupling strength. Then, by considering that the KSFR relation is hold and SU(3) symmetry ( $g_\omega = 3g_\rho$ ), the  $\rho$  meson channel amplitude at zero momentum becomes:

$$\mathcal{A}_{\gamma 3\pi}^{VMD(\rho)} = \frac{6e}{g_\omega} \frac{g_{\omega\rho\pi}^{eff} g_{\rho\pi\pi}}{m_\rho^2} = \frac{3}{2} \frac{e}{4\pi^2 f_\pi^3} = \frac{3}{2} \mathcal{A}_{\gamma 3\pi}^{WZW}, \quad (11)$$

that is three halves of the total amplitude from the Chiral anomaly [4]. Therefore, the contact term must account for the one half excess of the total amplitude. This was found by Rudaz [7] as a consistency requirement and by Cohen [8] as the one which satisfies axial Ward identities.

$$\mathcal{A}^c = \frac{6e}{g_\omega} g_{3\pi}^c = \frac{-1}{2} \mathcal{A}^{WZW} \quad (12)$$

this condition fixes the corresponding coupling to be:

$$g_{3\pi}^c = -\frac{g_{\rho\pi\pi}}{16\pi^2 f_\pi^3} = -47 \text{ GeV}^{-3}. \quad (13)$$

where we have made use of the relationship among the couplings as discussed above.

The previous analysis relies mainly on the consideration that the vector channel is saturated by the  $\rho$  meson, and upon matching VMD with the WZW anomaly the  $g_{\omega\rho\pi}$  coupling constant becomes an effective coupling which not only accounts for the  $\omega - \rho - \pi$  interaction but also all additional terms. In order to determine the truly  $g_{\omega\rho\pi}$  coupling, the radial excitations spectrum and the corresponding couplings are required. In the following we study a particular model to include these excitations and show their implications.

### III. ADDING THE RADIAL EXCITATIONS CONTRIBUTION.

To include the radial excitations of the  $\rho$  meson we make the following assumptions:

i) The relation  $(g_{\omega\rho\pi}g_\rho)/(g_{\omega V\pi}g_V) = 1$ , where  $V$  is any  $\rho$  radial excitation. That is, the radial excitation information makes no difference on the above combination of the couplings.

ii) SU(3) symmetry, which allows to relate  $g_\omega = 3g_\rho$ .

iii) KSFR-like relation for each  $V\pi\pi$  vertex, that is  $g_{V\pi\pi} = m_V/(\sqrt{2}f_\pi)$

iv) The spectrum of the radial excitations is given by:  $m_V^2 = m_\rho^2 n^2$  with  $n = 1, 2, \dots$ . The  $n = 1$  is by construction the  $\rho$  mass and for  $n = 2$  it is 1540 MeV to be compared with the  $\rho'(1450)$ .

As a first case we will consider the  $\omega \rightarrow \pi^0\gamma$  decay to identify the  $g_{\omega\rho\pi}$  and the  $\rho$  radial excitations effect. This process is clean in the sense that it is sensitive to the  $\rho$  radial excitations but not to the  $\omega$  ones. This fact allows to avoid any assumption on the  $\omega$  excitations. Then, the implications on the cases discussed in the previous section will be addressed.

### A. The $\omega \rightarrow \pi^0\gamma$ decay

The amplitude for this process is of the form:

$$\mathcal{M}_{\omega\pi\gamma} = i\epsilon_{\mu\nu\lambda\sigma}q_1^\mu\eta^\nu k_2^\lambda\epsilon^\sigma\mathcal{A}_{\omega\pi\gamma}, \quad (14)$$

Again, in this notation  $\mathcal{A}_{\omega\pi\gamma}$  encodes the model details. For this process, the VMD description, including the radial excitations, requires that at zero momentum:

$$\mathcal{A}_{\omega\pi\gamma} = e \sum_V \frac{g_{\omega V\pi}}{g_V}. \quad (15)$$

Now, using the above assumptions the global coupling can be set as:

$$\begin{aligned} \sum_V \frac{g_{\omega V\pi}}{g_V} &= \sum_V \frac{g_{\omega\rho\pi}g_\rho}{g_V^2} = \sum_V \frac{g_{\omega\rho\pi}g_\rho 2f_\pi^2}{m_V^2} \\ &= \frac{g_{\omega\rho\pi}g_\rho 2f_\pi^2}{m_\rho^2} \sum_n \frac{1}{n^2} = \frac{g_{\omega\rho\pi}g_\rho 2f_\pi^2}{m_\rho^2} \frac{\pi^2}{6} \end{aligned} \quad (16)$$

where the convergence on the series above is equal to  $\pi^2/6$ . Using this in the amplitude, we can compute the decay width

$$\Gamma(\omega \rightarrow \pi^0\gamma) = \frac{\alpha g_{\omega\rho\pi}^2}{432} \pi^4 f_\pi^2 m_\omega \left(\frac{m_\omega}{m_\rho}\right)^2 \left(1 - \frac{m_\pi^2}{m_\omega^2}\right)^3 \quad (17)$$

Using the experimental branching ratio for the process  $BR(\omega \rightarrow \pi^0\gamma) = 8.28 \pm 0.28\%$  [9], we obtain the following value for the individual  $\omega - \rho - \pi$  coupling

$$g_{\omega\rho\pi} = 8.2 \pm 0.2 \text{ GeV}^{-1} \quad (18)$$

### B. The $\pi \rightarrow \gamma\gamma$ decay

Lets us reconsider the  $\pi \rightarrow \gamma\gamma$  decay. It is important to explore the effect of the radial excitations in the relation

for the  $g_{\omega\rho\pi}^{eff}$ . In the VMD description, the coefficient in the amplitude, Eqn. (4), including an infinite sum of vector-like contributions  $V$ , becomes :

$$\mathcal{A}_{\pi\gamma\gamma}^{VMD+} = 2e^2 \sum_V \left( \frac{g_{V\omega\pi}}{g_\omega g_V} \right), \quad (19)$$

Using the assumptions about the couplings and masses of the vector mesons, it takes the following form:

$$\begin{aligned} 2e^2 \sum_V \frac{g_{V\omega\pi}}{g_\omega g_V} &= \frac{4e^2 g_{\omega\rho\pi} f_\pi^2}{3m_\rho^2} \sum_n \frac{1}{n^2} \\ &= \frac{2e^2 g_{\omega\rho\pi} \pi^2 f_\pi^2}{9m_\rho^2}. \end{aligned} \quad (20)$$

The consistency with the anomaly, Eqn. (4), requires that:

$$\begin{aligned} g_{\omega\rho\pi} &= \frac{9m_\rho^2}{8\pi^4 f_\pi^3} = 8.6 \text{ GeV}^{-1} \\ &= g_{\omega\rho\pi}^{eff} \frac{6}{\pi^2} \end{aligned} \quad (21)$$

which is consistent with the extracted value for the  $g_{\omega\rho\pi}$  coupling, Eqn. (18). In addition, if we take the value from Eqn. (18), the last relation for the effective coupling gives:

$$g_{\omega\rho\pi}^{eff} = 13.6 \pm 0.3 \text{ GeV}^{-1}, \quad (22)$$

we can observe that this value is also consistent with the corresponding one obtained in section II.A (Eqn. 7).

Note that  $g_{\omega\rho\pi}^{eff}$  in that case was defined as the one that resumed all the contributions in the  $\rho$  channel and made to agree with the anomaly. Thus, the above result tell us that the individual channel coupling is about 40% smaller than that value. It is trivial to check that by using this value when adding all the contributions we recover the amplitude of the previous section.

### C. The $\gamma^* \rightarrow 3\pi$ decay

Proceeding along the same lines, we can compute the contribution of the radial excitations. The amplitude for the vector channel is given by:

$$\begin{aligned} \mathcal{A}_{\gamma 3\pi}^{VMD+} &= \frac{6e}{g_\omega} \sum_V \frac{g_{V\omega\pi} g_{V\pi\pi}}{m_V^2} = 2e \frac{g_{\omega\rho\pi}}{m_\rho^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \\ &= e\pi^2 \frac{g_{\omega\rho\pi}}{3m_\rho^2} = \frac{3}{2} \mathcal{A}_{\gamma 3\pi}^{WZW} \end{aligned} \quad (23)$$

That is, by adding all the vector contributions we recover the result of the previous section for the  $\rho$ , but now corresponding to the full vector contributions.

The, now obvious, purely axial contribution in the contact term remains the same:

$$\mathcal{A}^c = \frac{-1}{2} \mathcal{A}_{\gamma 3\pi}^{WZW}, \quad (24)$$

Coupling	Process	Value
$ g_{\omega\rho\pi} $	$\omega \rightarrow \pi^0\gamma$ $\pi^0 \rightarrow \gamma\gamma$	$8.2 \pm 0.2 \text{ GeV}^{-1}$ $8.6 \text{ GeV}^{-1}$
$ g_{\omega\rho\pi}^{eff} $	$\pi^0 \rightarrow \gamma\gamma$	$14.2 \text{ GeV}^{-1}$
	$\omega \rightarrow \pi^0\gamma$ and $\pi^0 \rightarrow \gamma\gamma$	$13.6 \pm 0.3 \text{ GeV}^{-1}$
$g_{3\pi}^c$	$\gamma^* \rightarrow 3\pi$	$-47 \text{ GeV}^{-3}$
$g_{3\pi}^{re}$	$\gamma^* \rightarrow 3\pi$	$46 \text{ GeV}^{-3}$

TABLE I: Couplings

while the relationship for the  $\rho$  channel ( $n = 1$ ) now accounts for

$$\mathcal{A}_{\gamma 3\pi}^\rho = 2e \frac{g_{\omega\rho\pi}}{m_\rho^2} = \frac{e}{4\pi^2 f_\pi^3} \frac{9}{\pi^2} = \frac{6}{\pi^2} \frac{3}{2} \mathcal{A}_{\gamma 3\pi}^{WZW} \quad (25)$$

Since we have estimated all the vector contributions, the radial excitations contributions are just that with the  $\rho$  contribution removed:

$$\mathcal{A}_{\gamma 3\pi}^{re} = 2e \frac{g_{\omega\rho\pi}}{m_\rho^2} \left( \frac{\pi^2}{6} - 1 \right) \quad (26)$$

We can also treat this contribution as a contact diagram and estimate a magnitude for its effective coupling  $g_{3\pi}^{re}$ . The corresponding contact and radial excitations contributions can be written in an equivalent form as follows:

$$\frac{6eg_{3\pi}^{re}}{g_\omega} = 2e \frac{g_{\omega\rho\pi}}{m_\rho^2} \left( \frac{\pi^2}{6} - 1 \right), \quad (27)$$

from this equivalence we get

$$g_{3\pi}^{re} = \frac{g_\rho g_{\omega\rho\pi}}{m_\rho^2} \left( \frac{\pi^2}{6} - 1 \right) = 44 \text{ GeV}^{-3}. \quad (28)$$

Therefore, the remaining vector and axial contributions combine to account for a fraction of the total amplitude which is:

$$\mathcal{A}^{re + c} = \left( 1 - \frac{9}{\pi^2} \right) \mathcal{A}^{WZW} \quad (29)$$

Note that the global factor suppresses this contribution, which is in accordance with the individual effective couplings obtained above.

In table I we summarize the numerical results for the couplings in the different scenarios we have considered.

#### IV. DISCUSSION

The extraction of the  $g_{\omega\rho\pi}$  coupling is by nature made by indirect means. In this work we have pointed out that

the usually quoted value corresponds not to it but to another  $g_{\omega\rho\pi}^{eff}$  effective coupling which resumes not only the  $\rho$  meson effect but also all its additional radial excitation modes.

Here, we have explicitly added the radial excitations of the  $\rho$  meson, considering a particular form of the spectrum and relations among the couplings. First, we considered the  $\omega \rightarrow \pi^0\gamma$  decay to identify the single  $g_{\omega\rho\pi}$  and the  $\rho$  radial excitations effect. This process is clean in the sense that it is sensitive to the  $\rho$  radial excitations but not to the  $\omega$  ones. This fact allows to avoid any assumption on the  $\omega$  excitations. Certainly, the description used for all those excitations is model dependent. However, the low lying excitations are well approached and expected to be the dominant ones. Besides this model dependence, our description succeeds in exhibiting how each contribution came into the game while fulfilling general requirements like the agreement between the VMD and chiral anomaly descriptions. We obtained that the individual coupling is  $g_{\omega\rho\pi} = 8.2 - 8.6 \text{ GeV}^{-1}$ , which is about 40% smaller than the effective  $g_{\omega\rho\pi}^{eff}$ .

We have verified the consistency with the chiral approach in the  $\pi^0 \rightarrow \gamma\gamma$  and  $\gamma^* \rightarrow 3\pi$  processes. Besides the model dependence, our description succeeds in exhibiting how each contribution came into the game. In particular, we show that, for the  $\gamma^* \rightarrow 3\pi$  decay, the usual relation  $\mathcal{A}_{\gamma 3\pi}^{VMD+} = (3/2) \mathcal{A}_{\gamma 3\pi}^{WZW}$ , encodes all the vector contributions and not only the  $\rho$  meson one. Thus, the additional contact term is fully axial and fixed by the WZW anomaly. In addition, we have obtained that there is an almost exact (accidental) cancelation between the radial excitations and the contact term contributions. The relations here established are hold in the zero momentum limit. More elaborated assumptions would be required to explore in a reliable way the momentum dependence.

We would like to conclude by stressing that the usually neglected contributions from radial excitations may be relevant even though they can be very heavy. Therefore, consistency requirements like the ones here exhibited must be carefully considered.

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